C 63532 (Pages : 2) Name..................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CCSS)

Mathematics

MAT 2C 09—TOPOLOGY—I

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the questions
Each question carries 4 marks

1. Give examples of two topologies on a finite set X such that one is weaker than the other.
2. Prove that open balls in a metric space are open sets.
3. Define subspace of a topological space. Give an example of a non-trivial subspace of the set of real numbers with usual topology.
4. Define clopen set in a topological space. Give an example for a clopen set.
5. Prove that every open surjective map is a quotient map.
6. Prove that the property of being a discrete space is divisible.
7. Prove that every second countable space is Lindeloff.
8. Prove that a metric space is a T_2 space.
9. Prove that regularity is a hereditary property.
10. Prove that the projection functions are open.
11. Is the union of any two connected sets connected? Why?
12. Justify the terms 'box' and 'wall' geometrically for products of copies of the real-line.

($12 \times 4 = 48$ marks)

Turn over

Part B

Answer either A or B of each question
Each question carries 8 Marks

13. A. (a) Prove that the usual topology on the Euclidean plane R^2 is strictly weaker than the topology induced by lexicographic ordering.

(b) Define bounded set in a metric space Prove that in a metric space, every open ball is bounded.

B. (a) Determine the topology induced by a discrete metric on a set.

(b) If a space is second countable, prove that every open cover of it has a countable subcover.

14. A. (a) Prove that metrisability is a hereditary property.

(b) Prove that a second countable space always contains a countable dense subset.

B. (a) For any subset A of a space X, with usual notations prove that $\bar{A} = A \cup A'$.

(b) Define nearness relation on a set. Prove that there is a one-to-one correspondence between the set of topologies on a set and the set of all nearness relations on that set.

15. A. (a) Define hereditary topological property. Prove that the property of being a discrete space is hereditary.

(b) State and prove the Lebesgue covering lemma.

B. (a) Prove that every second countable space is first countable.

(b) Prove that every closed and bounded interval is compact.

16. A. (a) Prove that normality is a weakly hereditary property.

(b) Prove that a subset of X is a box if and only if it is the intersection of a family of walls. Also state and prove the corresponding property for a subset of X to be a large box.

B. (a) Prove that in a Hausdorff space, limits of sequences are unique.

(b) Prove that arbitrary product of T_1 spaces is T_1.

(4 × 8 = 32 marks)