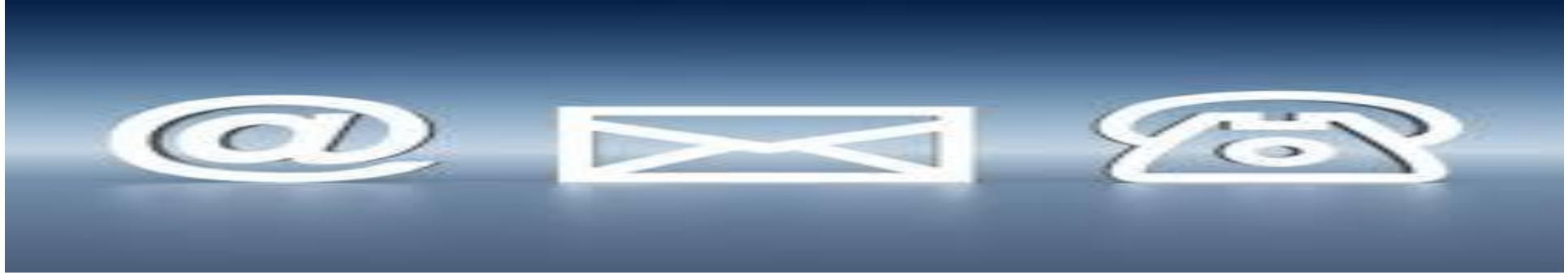


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# PHILOSOPHY OF MATHEMATICS

**Lecture 4**

One class is similar to another class if there is a bijection between them

i. e. a one to one correspondence

- The number of a class is the class of all those classes that are similar to it
- A number is anything which is the number of some class

# JOHN VON NEUMANN

0= Φ

1= {0}

2= {0,1}

3= { 0,1,2}

n+1= n∪ $\{n\}$

…

# INFINITY

# DAVID HILBERT

The infinite! No other question has ever moved profoundly the spirit of man; no other idea has so fruitfully stimulated his intellect; yet no other concept stands in greater need of clarification than that of the infinite.

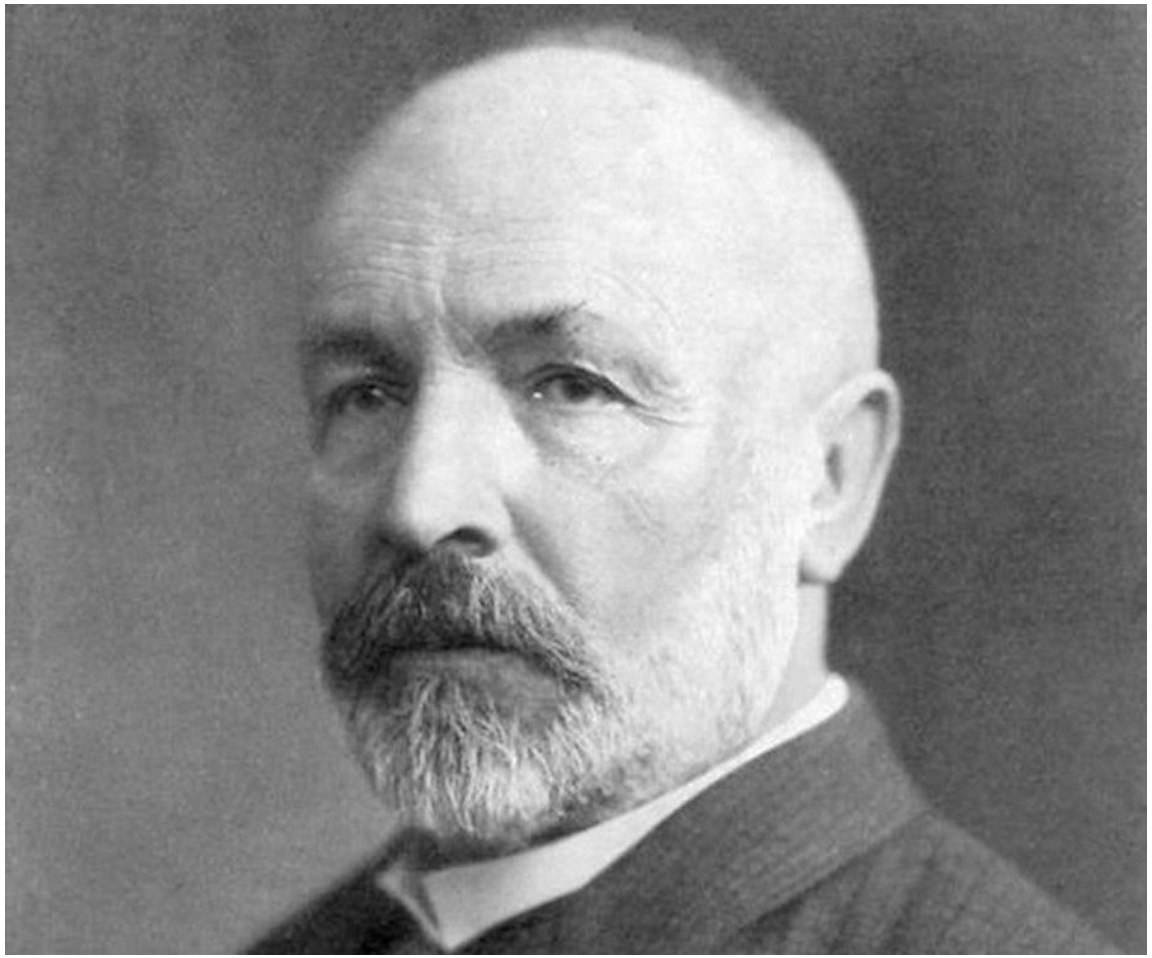
1921

# MARCUS AURELIUS

( Roman Emperor and Philosopher  121-180)

Infinity is a fathomless gulf, into which all things vanish

# CANTOR



# CANTOR

The essence of mathematics lies in its freedom

# CARDINAL NUMBERS

Cardinal number is a symbol assigned to a set in such a way that two sets are assigned the same symbol if and only if there is a bijection between them.

# CARL FRIEDRICH GAUSS

Infinity is only a figure of speech, meaning  a limit to which a certain ratios may approach as closely as desired, when others are permitted to increase indefinitely

# GALILEO GALILEI

Infinities and indivisibles transcends our finite understanding, the former on account of their magnitude, the latter because of their smallness; imagine what they are when combined

# H.POINCARE

Later generations will regard Mengen lehre as a disease from which one had recovered

ICM 1908 Paris

# L.KRONECKER

I don’t know what predominates in Cantor’s theory- philosophy or theology; but I am sure that there is no mathematics there

- “No one can chase us out of the paradise that Cantor has created for us”

David Hilbert

# IFININTESIMALS

An infinitesimal is a  number $\varepsilon$ such that

$|\varepsilon| < 1/n$ for every positive integer n.

Used by

Archimedes

Wilhelm Gottfried Leibniz

Does there exist nonzero infinitesimals?

# BISHOP GEORGE BERKELEY

The Analyst : or a Discourse Addressed to an Infidel Mathematician. Wherein it is Examined Whether the Object, Principles and Inferences of the Modern Analysis are More Distinctly Conceived, or More Evidently Deduced than Religious Mysteries and Points of Faith (1734) " he who can digest a second or third fluxion, a second or third difference , need not be squeamish about any point in divinity".

# BISHOP GEORGE BERKELEY

He argued that if the increment is a nonzero number, however small then it cannot be ignored and if it is zero then we cannot divide by it.

i) the nature of the objects studied in mathematics (ontology) and

ii) the knowledge of the truth of assertions about these objects (epistemology).

# ROGER PENROSE

English mathematical physicist, mathematician and philosopher of science.

IS MATHS DISCOVERED OR INVENTED?

- 'I certainly belong to the school of thought that it's discovery. Some of these have features which reveal deep truths which you had no conception of before.'

# G.H. HARDY

I believe that mathematical reality lies outside us, and our function is to discover or observe it and that the theorems which we prove, and which we describe grandiloquently as our ‘creations’ are simply our notes of our observations

**A Mathematician's Apology**

# G.H. HARDY

“ in some sense, mathematical truth is part of objective reality”

“ pure geometries are not pictures …… [of] the spatio-temporal reality of the physical world”

# P.W. BRIDGMAN

The distinguished Physicist

“ it is the merest truism, evident at once to unsophisticated observation, that mathematics is a human invention”

# EDWARD KASNER & JAMES NEWMAN

“ we have overcome the notion that mathematical truths have an existence independent and apart from our own minds. It is even strange to us that such a notion could ever have existed”

Do mathematical truth reside in the external world, there to be discovered by man, or are they man made inventions?

Does mathematical reality have an existence and validity independent of human species or is it merely a function of human nervous system?

# PLATONISM

Mathematical objects are real. Their existence is an objective fact, quite independent of our knowledge of them

Rene Thom

Kurt Godel

# PLATONISM

Platonism about mathematics (or *mathematical platonism*) is the metaphysical view that there are abstract mathematical objects whose existence is independent of us and our language, thought, and practices. Just as electrons and planets exist independently of us, so do numbers and sets. And just as statements about electrons and planets are made true or false by the objects with which they are concerned and these objects' perfectly objective properties, so are statements about numbers and sets. Mathematical truths are therefore discovered, not invented.

# PLATONISM

Mathematical platonism can be defined as the conjunction of the following three theses:

- **Existence**.
  There are mathematical objects.
- **Abstractness**.
  Mathematical objects are abstract.

  ( No spatio temporal existence)
- **Independence**.
  Mathematical objects are independent of intelligent agents and their language, thought, and practices.

# DAVID MUMFORD

Platonism is the belief that there is a body of mathematical objects, relations and facts about them that is independent of and unaffected by human endeavors to discover them.

# THE HORNED MOSES BY MICHELANGELO

# MICHELANGELO'S STONE

During the Italian Renaissance, Michelangelo Buonarroti, one of the greatest artists of all times, said that a good sculptor does not create a statue: he simply "takes it out" from the block of stone where the statue already lay hidden. A statue is already there, in its block of stone. The artists must simply expose it, carving away the redundant stone. The artist does not "create" the statue: he "finds" it.

# FORMALISM

D. Hilbert

There are no mathematical objects

Mathematics just consists of axioms, definitions and theorems

Mathematics is a game with symbols subject to certain rules

The formalist thesis is that mathematics is concerned with formal symbolic systems.

In the formalist thesis we have the axiomatic development of mathematics pushed to its extreme.

The establishment of the consistency of various parts

Proof theory

Hilbert & Bernays: Grundlagen der Mathematik

Vol. I 1934and Vol II in 1939

# CONSTRUCTIVISM

- The genuine mathematics is only what can be obtained by a finite construction.

# INTUITIONISM

- L.E.J. Brouwer
- L. Kronecker
- H. Poincare
- A.Heyting

# BROUWER, LUITZEN EGBERTUS JAN *(1881–1966)*

The founder of mathematical intuitionism, was born in 1881 in Overschie, near Rotterdam, the Netherlands. After attending schools in Medemblik, Hoorn, and Haarlem, he studied mathematics at the Municipal University of Amsterdam. He obtained his doctorate in 1907 for his thesis, *Over de Grondslagen der Wiskunde* (Amsterdam and Leipzig, 1907). He became *privaat-docent* at Amsterdam in 1909 and served as professor there from 1912 until his retirement in 1955. In the year that he became a professor he was elected to the Royal Dutch Academy of Sciences.

# A.HEYTING

**Intuitionism, An Introduction**

North- Holland Publishing Co., Amsterdam (1956)

Intuitionist thesis is that mathematics is to be built solely by finite constructive methods on the intuitively given sequence of natural numbers.

# LEOPOLD KRONECKER (1823 - 1891)

God made the integers, all else is the work of man

What good your beautiful proof on [the transcendence of] π? Why investigate such problems, given that irrational numbers do not even exist?

For Intuitionists, the law of excluded middle holds for finite sets but should not be employed when dealing with infinite sets.

According to Brouwer the laws of logic emerged at the time in man's evolution when he had a good language for dealing with finite sets of phenomena; he then later made the mistake of applying these laws to the infinite sets of mathematics, with the result that antinomies arose.

# LOGICISM

Richard Dedekind

Gottlob Frege

Bertrand Russell ( 1872-1970)

Alfred North Whitehead ( 1861-1947)

# LOGICISM

What is the essence of mathematics?

Mathematics is essentially logic

Mathematics is an extension of logic.

Mathematics is reducible to logic

Axioms + Logic → mathematics

Principia Mathematica, B.Russell& A.N.Whitehead

A detailed reduction of Mathematics to logic

- Wittgenstein
- Ramsey
- Langford
- Carnap
- Quine

etc.

# ISMS

Platonism • Formalism • Logicism • Intuitionism / Constructivism • Structuralism • Fictionalism • Naturalism-Empiricism • Social Constructivism

# F.ENGELS

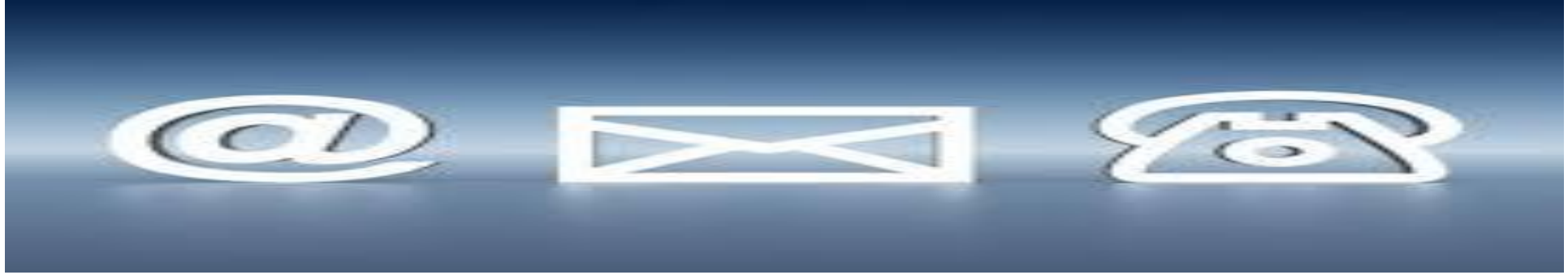
Mathematics is an abstract science which is concerned with the creations of thought, even though they are reflections of reality

**Dialectics of Nature**

# WHAT IS MATHEMATICS?

Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.

Bertrand Russell, Mysticism and Logic (1917) ch. 4

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