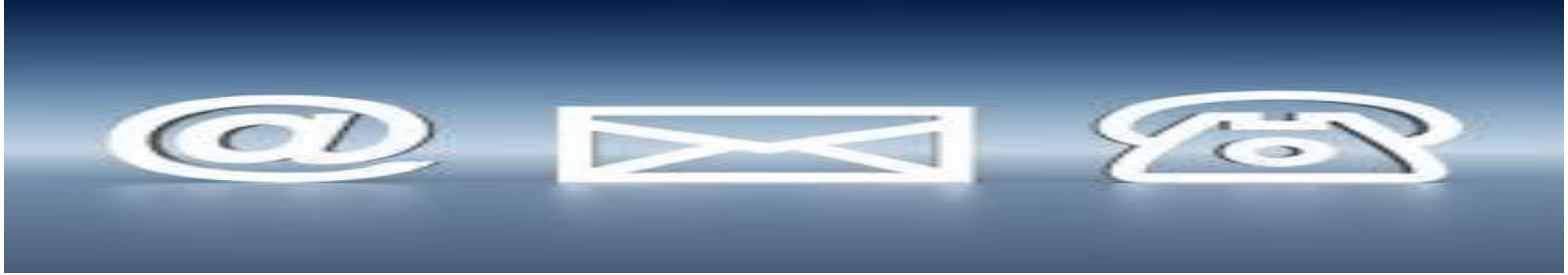

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PHILOSOPHY OF MATHEMATICS

Lecture 2

TWO APPROACHES

1.Constructive

2. Analytic

FOUNDATIONS OF MATHEMATICS

LOGICISM

Logicism is a programme in the philosophy of mathematics, comprising one or more of the theses that — for some coherent meaning of 'logic' — mathematics is an extension of logic, some or all of mathematics is reducible to logic, or some or all of mathematics may be modelled in logic.[1] Bertrand Russell and Alfred North Whitehead championed this programme, initiated by Gottlob Frege and subsequently developed by Richard Dedekind and Giuseppe Peano.

Mathematics is an extension of logic

LOGIC

LAW OF EXCLUDED MIDDLE

Every statement is either true or false but not both.

BUDHISM

Nagarjuna (2nd or 3rd Century C.E.):

Tetralemma

Anything is either true, or not true or both true and not true or neither.

NAGARJUNA

JAINISM

- Syad Vada
- Saptabhangi
- Bhadrabahu (c. 433–357 BCE).

SAPTABHANGI

- Arguably, it (that is, some object) exists (*syad asty eva*).
- Arguably, it does not exist (*syan nasty eva*).
- Arguably, it exists; arguably, it doesn't exist (*syad asty eva syan nasty eva*).
- Arguably, it is non-assertible (*syad avaktavyam eva*).
- Arguably, it exists; arguably, it is non-assertible (*syad asty eva syad avaktavyam eva*).
- Arguably, it doesn't exist; arguably, it is non-assertible (*syan nasty eva syad avaktavyam eva*).
- Arguably, it exists; arguably, it doesn't exist; arguably it is non-assertible (*syad asty eva syan nasty eva syad avaktavyam eva*).

MANY VALUED LOGICS

In logic, a **many-valued logic** (also **multi-** or **multiple-valued logic**) is a propositional calculus in which there are more than two truth values. Traditionally, in Aristotle's logical calculus, there were only two possible values (i.e., "true" and "false") for any proposition. Classical two-valued logic may be extended to ***n*-valued logic** for *n* greater than 2. Those most popular in the literature are three-valued (e.g., Łukasiewicz's and Kleene's, which accept the values "true", "false", and "unknown"), the finite-valued (finitely-many valued) with more than three values, and the infinite-valued (infinitely-many valued), such as fuzzy logic and probability logic.

FUZZY LOGIC

L.A. ZADEH

IGNORANCE

BERTRAND RUSSELL
1950

The central problem of our age is how to act decisively in the absence of certainty.

IGNORANCE AND KNOWLEDGE

Without ignorance, there would be no knowledge, without knowledge, there would be no ignorance.

IGNORANCE

- The representation and management of ignorance has become a more central concern in older fields such as cognitive psychology, economics, management science and sociology of organizations.

- It is virtually a hot topic in newer fields such as expert systems, artificial intelligence and risk analysis.

AGNOTOLOGY

- The study of Ignorance

Agnotology: The Making and Unmaking of Ignorance

Robert Proctor, Londa Schiebinger, Londa L. Schiebinger

Stanford University Press, 2008

IGNORANCE AND UNCERTAINTY

MICHAEL SMITHSON

Springer Verlag 1989

TAXONOMY OF IGNORANCE

Michael Smithson 1988

Ignorance and Uncertainty

IGNORANCE

- ERROR

  The passive state of ignorance

- IRRELEVANCE

  Consequence of the act of ignoring

ERROR

- DISTORTION

  Distorted views

- INCOMPLETENESS

  Incomplete views

DISTORTION

- CONFUSION

  Distortion in degree

  Due to bias or inaccuracy

- INACCURACY

  Wrongful substitution in kind

INCOMPLETENESS

- UNCERTAINTY

  Incompleteness in degree

  Partial information

- ABSENCE

  Incompleteness in kind

  Gaps

UNCERTAINTY

- VAGUENESS

Range of possible values on a continuum

- PROBABILITY

chance

- AMBIGUITY

Finite number of distinct possibilities

Food is hot (temperature or spicyness)

VAGUENESS

- Fuzzyness

Fine grade distinctions and blurry boundaries

Example: Dark

- Non-specificity

  Example: near the school

VAGUENESS

- The concept of a set is fundamental for the whole of mathematics.

- Mathematicians require that all mathematical notions including that of a set must be exact.

- How ever philosophers and recently computer scientists as well as other researchers have become interested in vague concepts

- Almost all concepts we are using in natural languages are vague

- Probability Theory
- Various types of Generalised Set Theories

SET THEORIES

- Fuzzy Set Theory
- Rough Set Theory
- Soft Set Theory
- Multi Sets
- Nonstandard Set Theory
- Alternative Set Theory

FOUR COLOUR THEOREM

- In mathematics, the **four color theorem**, or the **four color map theorem**, states that, given any separation of a plane into contiguous regions, producing a figure called a *map*, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color. *Adjacent* means that two regions share a common boundary curve segment, not merely a corner where three or more regions meet.[1] It was the first major theorem to be proved using a computer. Initially, this proof was not accepted by all mathematicians because the computer-assisted proof was infeasible for a human to check by hand.[2] Since then the proof has gained wide acceptance, although some doubters remain.[3]
- The four color theorem was proved in 1976 by Kenneth Appel and Wolfgang Haken after many false proofs and counterexamples (unlike the five color theorem, a theorem that states that five colors are enough to color a map, which was proved in the 1800s). To dispel any remaining doubts about the Appel–Haken proof, a simpler proof using the same ideas and still relying on computers was published in 1997 by Robertson, Sanders, Seymour, and Thomas. Additionally, in 2005, the theorem was proved by Georges Gonthier with general-purpose theorem-proving software.

THE CONCEPT OF NUMBERS

NON NEGATIVE INTEGERS

NATURAL NUMBERS

ARITHMETISATION OF MATHEMATICS

All the traditional pure mathematics can be derived from the natural numbers

AXIOMATIC THEORY OF NATURAL NUMBERS

Reduce number theory to the smallest set of primisses and undefined terms from which it could be derived

This was accomplished by Peano

Three primitive ideas in Peano’s arithmetic are

0, number, successor

AXIOMATIC METHOD

Why?

How?

Axioms

Undefined concepts

GIUSEPPE PEANO

Giuseppe Peano (/piˈɑːnoʊ/;[1] Italian: [dʒuˈzɛppe peˈaːno]; 27 August 1858 – 20 April 1932) was an Italian mathematician and glottologist. The author of over 200 books and papers, he was a founder of mathematical logic and set theory, to which he contributed much notation. The standard axiomatization of the natural numbers is named the Peano axioms in his honor. As part of this effort, he made key contributions to the modern rigorous and systematic treatment of the method of mathematical induction. He spent most of his career teaching mathematics at the University of Turin. He also wrote an international auxiliary language, Latino sine flexione ("Latin without inflections"), which is a simplified version of Classical Latin. Most of his books and papers are in Latin sine flexione, others are in Italian.

GIUSEPPE PEANO

PEANO'S AXIOMS

1. 0 is a number
2. The successor of any number is a number
3. No two numbers have the same successor
4. 0 is not the successor of any number
5. Any property which belongs to 0, and also to the successor of every number which has the property, belongs to all numbers

   (Induction Axiom)

PRINCIPLE OF MATHEMATICAL INDUCTION

- Induction
- Deduction

INDUCTION AND MATHEMATICS

The passage from particular assertions to general ones is called induction

INDUCTION FAILS

FERMAT PRIMES

F_n = 2^2^n+1

F_0=3 prime

F_1= 5 prime

F_2=17 prime

F_3=257 prime

F_4=65537

F_5 = 4294967297 = 641x 6700417 is not a prime

641 is a factor

Euler 1732

G.W.LEIBNITZ

n^3-n is divisible by 3 for all positive integers n

n^5-n is divisible by 5 for all positive integers n

n^7-n is divisible by 7 for all positive integers n

But 2^9-2=510 is not divisible by 9

INDUCTION FAILS

$P(x)=x^2+x+41$

x	P(x)	
1	43	
2	47	are primes
3	53	
4	61	
5	71	
6	83	
7	97	
8	113	
9	131	
10	151	
.....	......	

INDUCTION FAILS

X=40

P(x)= x^2+x+41

$P(40) = 40^2 +40+41= 40^2 +40+40+1$

$= 40^2 +2x\ 40 +1$

$=(40+1)^2= 41^2$

ANOTHER EXAMPLE

991n2 +1 is not a perfect square

for all n up to and including

n= 12055735790331359447442538766

But is a perfect square for

n= 12055735790331359447442538767

29 digits!

INDUCTION FAILS

D.A. Grave

For any prime p less than 1000

2^{p-1} is not divisible by p^2

But 1093 is a prime

2^{1093-1} is divisible by 1093^2

INDUCTION FAILS

An assertion may be valid in an entire series of particular cases, and at the same time invalid in general

PRINCIPLE OF MATHEMATICAL INDUCTION

P(n) for n= 1,2,3…

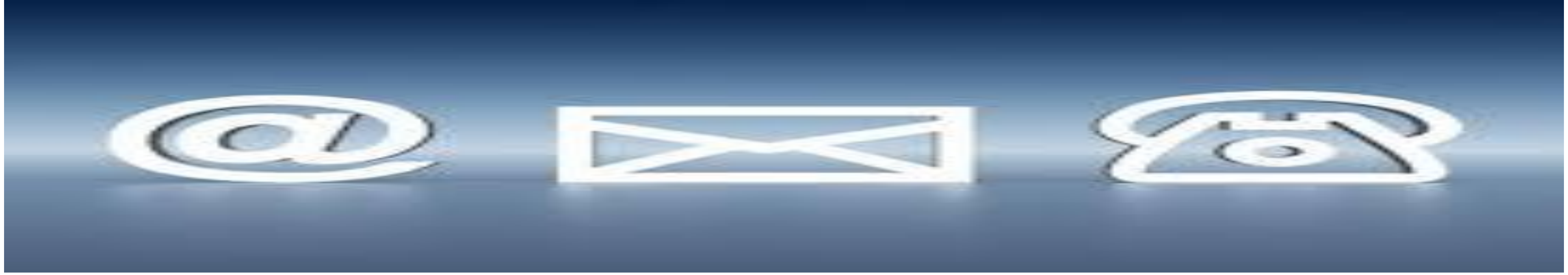
P(1) is true

P(k) implies P(k+1) for all k

Then P(n) is true for all n=1,2,3…

WELL ORDERING PROPERTY

Any nonempty set of positive integers contains a smallest element



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